# APPENDIX A

The MATHEMATICA code to calculate the dependence of film color on thickness is reasonably succinct, and I shall therefore include it for reference. The code which defines to first order the reflected intensity from a thin film is (see equation (6.8))

$$\texttt{Ir}[\lambda\_, \texttt{t}\_, \theta\_] := \texttt{4 Ii R Sin}\left[\frac{2\pi}{\lambda}\,\mu\,\texttt{h Cos}[\theta]\right]^{2} \tag{6.11}$$

Since we are not interested here with the specific magnitude of the reflected intensity, but only in a comparison between the intensities of different wavelengths, we set the reflectance R to 1. We also approximate the index of refraction of the solution as that of bulk water,

$$\texttt{R} = 1; \mu = 1.33; \tag{6.12}$$

The response of the various opsins to light is then approximated by

$$\texttt{k}[\lambda\_, \texttt{j}\_] := \frac{1}{\sigma\sqrt{\pi}}\texttt{Exp}\left[-\left(\frac{\lambda - \lambda\texttt{c}[[\texttt{j}]]}{\sigma}\right)^{2}\right] \tag{6.13}$$

where

$$\sigma = 30 * 10^{-9}; \tag{6.14}$$

denotes the width of the response curve in meters and

$$\lambda\texttt{c} = \{575,\ 535,\ 445\}\ 10^{-9}; \tag{6.15}$$

holds the peak response wavelengths of the red, green, and blue opsins, respectively. Calculation of the response to uniform white light is achieved by simply setting the incident intensity `Ii` constant. An alternative is to construct an appropriate form for the incident light from known atomic emission spectra. We shall use sodium as an example due to its relative simplicity,

$$\begin{aligned}&\texttt{sodiumlines} = \\ &\quad \{\{3302, 1.2\}, \{5890, 80\}, \{5896, 40\}\}; \\ &\sigma\texttt{lw} = 6 * 10^{-9};\end{aligned} \tag{6.16}$$

where the first term in each pair denotes the wavelength in angstroms and the second term the relative intensity. We also define a line width $\sigma$`lw` which is set to the smallest value that will yield reliable results during numerical integration. The actual spectrum is then constructed by mapping a Gaussian to each of the spectral lines and summing the result,

$$\begin{aligned}&\texttt{Ii} = \texttt{sodium} = \texttt{N}[ \\ &\quad \texttt{Plus}@@ \\ &\qquad \left(\frac{\#[[2]]}{\sigma\texttt{lw}\sqrt{\pi}}\ \texttt{Exp}\left[-\left(\frac{\lambda - \#[[1]] * 10^{-10}}{\sigma\texttt{lw}}\right)^2\right]\ \& \right.\\ &\qquad\quad \left. /@\ \texttt{sodiumlines}\right) \\ &\quad ];\end{aligned} \tag{6.17}$$

Determination of the relative contributions to the color from each of the opsins is accomplished by numerical integration over the visible spectrum for each of the ocular response curves,

$$
\begin{aligned}
&\texttt{GetColor}[\texttt{t}\_, \theta\_] := \\
&\quad \texttt{Table}[ \\
&\qquad \texttt{NIntegrate}[\, \texttt{k}[\lambda, \texttt{j}]\ \texttt{Ir}[\lambda, \texttt{t}, \theta], \\
&\qquad\quad \{\lambda, 300 * 10^{-9}, 700 * 10^{-9}\}], \\
&\quad \{\texttt{j}, 1, 3\}]
\end{aligned} \tag{6.18}
$$

The first term of the resulting three element list corresponds to red, the second to green, and the last to blue. The numbers, however, are not, as yet, scaled to the $[0, 1]$ range required by the `RGBColor` function. This is to allow for different methods of normalization. For example, if the relative intensity between color bands is of primary importance, the RGB values for the entire thickness range can be rescaled by the maximal value of the entire set (useful for visualizing results from a monochromatic light source). Or, if an accurate determination of color is most important, each RGB triplet can be rescaled by its individual maximum (particularly useful for observation under nearly white light, or a complex spectrum such as xenon).

$$\texttt{T}[\texttt{x}\_] := \texttt{Transpose}[\texttt{x}] \tag{6.19}$$

We first make a kind of shorthand notation for a function we will use repeatedly, then set the thickness range and resolution for which we wish to calculate the color of reflected light,

$$\texttt{tmin} = 10^{-8};\ \texttt{tmax} = 6 * 10^{-6};\ \texttt{tstep} = 10^{-8}; \tag{6.20}$$

The result is placed in a table which contains both the color information and a suitable `Graphics` object,

$$
\begin{aligned}
&\texttt{colors} = \texttt{Table}[\\
&\quad \{\ \texttt{GetColor}[\texttt{t}, 0],\\
&\qquad \texttt{Rectangle}[\{\texttt{t}, 0\}, \{\texttt{t} + \texttt{tstep}, 1\}]\\
&\quad \},\\
&\quad \{\texttt{t},\ \texttt{tmin},\ \texttt{tmax},\ \texttt{tstep}\}\\
&];
\end{aligned}
\tag{6.21}
$$

The final output involves applying the `RGBColor` function to the color triplets after a suitable rescaling, and then display using `Show`,

$$
\begin{aligned}
&\texttt{Show}[\\
&\quad \texttt{Graphics}/@\ \texttt{T}[\ \{\\
&\qquad \texttt{Apply}[\texttt{RGBColor}, \frac{\texttt{T}[\texttt{colors}]\ [[1]]}{\texttt{Max}[\ \texttt{T}[\texttt{colors}]\ [[1]]\ ]}, \{1\}],\\
&\qquad \texttt{T}[\texttt{colors}]\ [[2]]\\
&\quad \}],\\
&\quad \texttt{Frame} \rightarrow \texttt{True}\\
&]
\end{aligned}
\tag{6.22}
$$